

# Filling factors and Braid group

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## Abstract

We extract the Braid group structure of a recently derived *hierarchy scheme* for the *filling factors* proposed by us which related the *Hausdorff dimension*,  $h$ , to the *statistics*,  $\nu$ , of the collective excitations in the context of the Fractional Quantum Hall Effect ( FQHE ).

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In a recent paper [1], following ideas established in [2], we have considered the *Hausdorff dimension*,  $h$ , as a parameter which classifies the *equivalence classes* of the *collective excitations* which occurs in the context of the FQHE. The elements of each equivalence class are the *filling factors or statistics*,  $\nu$ , which characterize the collective excitations manifested as quasiholes or quasiparticles [3]. Now, we propose rules of composition for these elements such that we extract a *Braid group* structure from this new hierarchy scheme for the filling factors. This scheme takes into account the intervals of definition of spin,  $s$ , for fractional spin particles, which are related to the Hausdorff dimension in such way that we can *predict* for what values of  $\nu$  FQHE can be observed [2]. For a given value of  $h$ , defined as  $1 < h < 2$ , we obtain  $\nu$  as (  $i$  means a specific interval ):

$$\begin{aligned}
h_1 &= 2 - \nu, & 0 < \nu < 1; & & h_2 &= \nu, & 1 < \nu < 2; \\
h_3 &= 4 - \nu, & 2 < \nu < 3; & & h_4 &= \nu - 2, & 3 < \nu < 4; \\
h_5 &= 6 - \nu, & 4 < \nu < 5; & & h_6 &= \nu - 4, & 5 < \nu < 6; \\
h_7 &= 8 - \nu, & 6 < \nu < 7; & & h_8 &= \nu - 6, & 7 < \nu < 8; \\
h_9 &= 10 - \nu, & 8 < \nu < 9; & & h_{10} &= \nu - 8, & 9 < \nu < 10; \\
h_{11} &= 12 - \nu, & 10 < \nu < 11; & & h_{12} &= \nu - 10, & 11 < \nu < 12; \\
h_{13} &= 14 - \nu, & 12 < \nu < 13; & & h_{14} &= \nu - 12, & 13 < \nu < 14; \\
h_{15} &= 16 - \nu, & 14 < \nu < 15; & & h_{16} &= \nu - 14, & 15 < \nu < 16; \\
h_{17} &= 18 - \nu, & 16 < \nu < 17; & & h_{18} &= \nu - 16, & 17 < \nu < 18; \\
h_{19} &= 20 - \nu, & 18 < \nu < 19; & & h_{20} &= \nu - 18, & 19 < \nu < 20; \\
h_{21} &= 22 - \nu, & 20 < \nu < 21; & & h_{22} &= \nu - 20, & 21 < \nu < 22; \\
h_{23} &= 24 - \nu, & 22 < \nu < 23; & & h_{24} &= \nu - 22, & 23 < \nu < 24; \\
h_{25} &= 26 - \nu, & 24 < \nu < 25; & & h_{26} &= \nu - 24, & 25 < \nu < 26; \\
&& & & & & \text{etc.}
\end{aligned} \tag{1}$$

In this way,  $h_i$ , classifies the excitations in equivalence classes, whose elements are the statistics,  $\nu$ . On the other hand, we know that for path integration on multiply connected spaces [4], we need to assign different weights,  $\chi$ , to homotopically disconnected paths. So, in terms of our scheme, as we just said,  $h_i$ , labels these classes, which form a group, the fundamental group  $\Pi_1(\mathcal{P})$ , with elements,  $\chi = \exp \{i\nu\varphi\}$ . We also have that, for the Abelian representation of the group, the weights satisfy the constraints [4]:

$$\begin{aligned}
|\chi \{ \nu_i \}_{h_i}| &= 1, \\
\chi \{ \nu_i \}_{h_i} \chi \{ \nu_j \}_{h_j} &= \chi \{ \nu_i \circ \nu_j \}_{h_{i \circ j}}.
\end{aligned} \tag{2}$$

The weights,  $\chi$ , represent a phase which we assign to the paths that contribute to the propagator:

$$\mathcal{K}(q', t'; q, t) = \sum_{\{ \nu \}_h} \chi \{ \nu \}_h \mathcal{K}^{\{ \nu \}_h}(q', t'; q, t). \tag{3}$$

We observe that, for the composition law, for example, with  $\nu_1 + \nu_2$ , we do not consider integers since  $\nu$  can only be a rational number with odd denominator ( see again the intervals

of  $\nu$ ). On the other hand, we verify that some compositions with elements of the same class give another element either in or out of the class. The same occurs for the distinct classes (this can be, perhaps, a non-Abelian manifestation of the group). But this does not matter, because we only consider paths into within the same class for the path integration. For example, consider the classes:

$$\begin{aligned} & \left\{ \frac{2}{3}, \frac{8}{3}, \dots \right\}_{h=\frac{4}{3}}; \quad \left\{ \frac{3}{5}, \frac{7}{5}, \dots \right\}_{h=\frac{7}{5}}; \\ & \left\{ \frac{5}{9}, \frac{13}{9}, \dots \right\}_{h=\frac{13}{9}}; \quad \left\{ \frac{4}{7}, \frac{10}{7}, \dots \right\}_{h=\frac{10}{7}}. \end{aligned} \quad (4)$$

If we compose,  $\left\{ \frac{2}{3} \right\}_{h=\frac{4}{3}}$  with  $\left\{ \frac{3}{5} \right\}_{h=\frac{7}{5}}$  we get  $\left\{ \frac{19}{15} \right\}_{h=\frac{19}{15}}$ ; if we compose  $\left\{ \frac{2}{3} \right\}_{h=\frac{4}{3}}$  with  $\left\{ \frac{8}{3} \right\}_{h=\frac{4}{3}}$  we get  $\left\{ \frac{10}{3} \right\}_{h=\frac{4}{3}}$ ; if we compose  $\left\{ \frac{3}{5} \right\}_{h=\frac{7}{5}}$  with  $\left\{ \frac{7}{5} \right\}_{h=\frac{7}{5}}$  we get  $\{2\}$  which is not defined, etc. All of this, expresses the intricacy of the anyonic model for such a physical phenomenon as the FQHE.

Now, a point of theoretical interest: Under the exchange of two quasiholes [4] we have that the factor  $(z_\alpha - z_\beta)^\nu$  of the Laughlin function, with  $\nu = \frac{1}{m} + 2p_1$ , where  $m = 3, 5, 7, \dots$  and  $p_1$  is a positive integer, produces the condition on the phase:

$$\exp \left\{ i\pi \left( \frac{1}{m} + 2p_1 \right) \right\} = \exp \left\{ i\pi \frac{1}{m} \right\}. \quad (5)$$

Observe that, for  $m = 3$  and  $p_1 = 1$ , if we take into account our formulas for  $h_i$ , we get:  $\left\{ \frac{1}{3}, \frac{7}{3} \right\}_{h=\frac{5}{3}}$ ; for  $m = 5$ ,  $p_1 = 1$ , we get  $\left\{ \frac{1}{5}, \frac{11}{5} \right\}_{h=\frac{9}{5}}$ ; for  $m = 7$ ,  $p_1 = 1$ , we get  $\left\{ \frac{1}{7}, \frac{15}{7} \right\}_{h=\frac{13}{7}}$ , etc. What does this tell us? This simply confirms our hierarchy scheme for the filling factors: *the Hausdorff dimension classifies the collective excitations into equivalence classes and, in fact, the weights are elements of the Braid group*.

Another point, which we can touch on is, with respect to the largest charge gaps, these are observed for  $\nu = \frac{1}{3}$  and  $\nu = \frac{2}{3}$  [5]. Second in our scheme,  $\left\{ \frac{1}{3} \right\}_{h=\frac{5}{3}}$  and  $\left\{ \frac{2}{3} \right\}_{h=\frac{4}{3}}$ , they are in different classes but the collective excitations for one case and other one is like a *quasi-bosonic regime*,  $h \sim 2$ , and a *quasi-fermionic regime*,  $h \sim 1$ , respectively. Thus, the interacting planar electron system presents this subtle behavior for all filling factors. Of course, this explains why some series of fractions are more favourable than others. Now, a last comment, which reinforces our classification of anyonic excitations, observe the mirror symmetry behind the construction of  $h$ .

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